Managerial Ontologies and Meaningfulness

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Two Events of 1946

- Appearance of ENIAC, the first large scale general purpose computer.
- Publication of S. S. Stevens paper, “On the theory of scales of measurement,” *Science*, *103*, 677–680.
- ENIAC led to today’s massive digital databases, computer modeling and simulation. Stevens’ paper signals the beginning of modern measurement theory.
- Are these developments related?

A Common Thread?

- Alexei Nemov (Olympic high bar routine)
- Average happiness
- BCS (Bowl Championship Series) rankings
- CAPPS II
- GPA (Grade point average)
- Managerial ontologies
- Rating agency surveillance
- Sovereign and corporate risk

Theories of Measurement

- *Representational*: Measurement is a mapping between an empirical (relational) system and a numerical (relational) system. (Model/Understand)
- *Operational*: "In general we mean by any concept nothing more than a set of operations; *the concept is synonymous with the corresponding set of operations*." (Bridgman) (Blackbox/Predict)
- *Classical*: Measurement discovers numerical properties or attributes of reality.
- List not exhaustive. Here we are concerned primarily with representational measurement theory.

Measurement Theories Example

"Take the case of scores on a scale measuring preference for one or two alternatives. Representational measurement theory will assign people to positions (and hence numbers) on the scale and will assert that only ordinality applies and can apply. In contrast, classical theory may assert that there is an underlying quantitative variable, but that the scale is but a poor measure of it (no doubt, with an unknown non-linear relationship to it). Operational theory will define this particular type of 'preference' as being the number that emerges from the exercise." (Hand)

Scale Types

Definition 1. Let A be a set. A real-valued *scale* $S = S(A,\mathbf{R})$ is a set of functions from A to the real numbers **R**. Elements of S are called *representations*.

Definition 2. Let S be a scale on A, then the *image* of S is the set

$$\mathrm{Im}S = \{ f(x) \mid x \in A \text{ and } f \in S \}.$$

Definition 3. A group G is a *scale group* of a scale S if G acts on ImS through composition of functions and there exists an $h \in S$ such that

$$S = G_h = \{ g \circ h \mid g \in G \}.$$

The *scale type* of a scale S refers to the scale group with orbit S. The functions in a scale group are called *admissible transformations*.

Partial Taxonomy of Scale Types

Scale	Basic empirical operations	Mathematical group	Permissible statistics (invariative)
Nominal	determination of equality	permutation group	number of cases mode
Ordinal	determination of greater or less	isotonic group	median percentiles
Interval	determination of equality of differences	affine or general linear group	mean standard deviation product-moment correlation
Ratio	determination of equality of ratios	similarity group	coefficient of variation
Absolute		trivial group	probability

Intuitive Meaningfulness

Definition 4. A statement involving numerical representations is *meaningful* if and only if its truth or falsity is unchanged under admissible transformation of all representations in the statement.

The basic idea behind the notion of meaningfulness is to guard against inferential artifacts of the numerical system being mistaken for valid inferences about the measured empirical system.

Stevens' definition of scale types and meaningful or permissible statistics led to an acrimonious debate in psychology: psychologists could now measure, but some felt that they were no longer permitted to calculate. The concept of meaningfulness has not been applied extensively outside the psychological disciplines.

Meaningless Statements

- The statement, “Yesterday the high was 30 degrees, today the high is 60, so it is twice as warm today as yesterday,” is meaningless since the admissible affine transformation $C = 5/9\,(F - 32)$ renders the statement false.
- Shortest paths (Roberts): A network has vertices *x*, *a*, *y* and arcs *ax* with weight 2, *ay* with weight 4, and *xy* with weight 40. The statement, “the shortest path from *x* to *y* is the path *xay*,” is meaningless for interval scale type weights since the admissible affine transformation $f(w) = 2w + 70$ results in the path *xy* having the shortest length.

Average Happiness

- Consider happiness scores for two groups of three people -- Group A, (2,2,2); Group B, (1,1,3) -- where 3 ≡ very happy, 2 ≡ moderately happy, 1≡ not happy.
- Is it meaningful to assert that group A's average happiness is greater than group B's (2.000 > 1.667)?
- The answer depends on the scale type of the happiness scores. Assuming ordinal scale type happiness scores, the admissible monotone transformation, 1 → 1.6, 2 → 2, and 3 → 3, results in group A being less happy on average than group B (2.000 < 2.061).
- A meaningful average requires that the happiness scores be interval scale types.

Geometric Mean

A function *u* satisfies *agreement* if $u(x,x,\ldots,x)=x$.

For ♦ a permutation of { 1, 2,..., *n* }, *u* satisfies *symmetry* if

$$u(x_1,x_2,\ldots,x_n)=u\left(x_{\sigma(1)},x_{\sigma(2)},\ldots,x_{\sigma(n)}\right).$$

Theorem. (Aczel and Roberts) Suppose f_1, f_2, ... , f_n are representations of ratio scale type with independent units (i.e., the admissible transformations g_i ∈ G_i may be different similarity transformations) and *u* is a merging function which satisfies agreement and symmetry. Then it is meaningful to assert, for any ♋ > 0 and any *a*, *b* ∈ *A*, that

$$u\left(f_1(a),f_2(a),\ldots,f_n(a)\right)=\alpha\, u\left(f_1(b),f_2(b),\ldots,f_n(b)\right),$$

if and only if *u* is the geometric mean.

Geometric Mean Example

Example. Suppose f_i (*x*) is the rating by the *i*-th expert of the importance of alternative *a* and *u* is an aggregate of group importance. If we assume that the expert ratings are independent ratio scales, then the geometric mean, not the arithmetic mean, provides the only meaningful synthesis of group importance.

While it is "'folklore' that comparison of arithmetic means is often not meaningful, while comparison of geometric means is," the remarkable implication of the previous theorem is that "no other merging functions are acceptable." (Aczel and Roberts)

Merging Ordinal Scales

Let $\boldsymbol{x} = \{ x_1,\ldots,x_n \}$ denote a vector (complex) of independent ordinal scales.

Theorem. (Osborne, Kim) If a continuous function *u* maps independent ordinal scales into an ordinal scale, then either *u* is a constant function or $u(\mathbf{x}) = h(x_i)$ for some $i \in \{1,\ldots,n\}$ where *h* is a continuous, strictly monotonic function.

Theorem. (Osborne, Kim) If a continuous function *u* maps independent ordinal scales into a ratio, interval, or log-interval scale, then *u* is a constant function.

Ovchinnikov has shown that the only (comparison) meaningful merging functions for ordinal scales are order statistics.

Intermixed Scale Types

- Databases involved in management decision making and ontology construction are often of intermixed scale type. However, there has been little investigation of meaningful merging of intermixed scale type data.
- Osborne observed that the lowest scale type in a complex determines the effective scale type of the complex.
- Analyzing data with respect to Osborne's lowest common denominator scale type is inefficient or losy as it ignores structure contained in higher scale type components.

Meaningful Methodologies

- Optimal scaling based algorithms including, correspondence analysis, nonlinear principal components analysis (PRINCALS), and nonmetric multidimensional scaling.
- Hybrid approaches with operational and representational aspects. Includes above and methods such as Abelson and Tukey's procedure for ANOVA on ordinal scale data.
- Construct validity. Measure validation based on consistency with other variables and reproducibility.
- Psychophysical techniques including magnitude estimation and cross modality matching (Stevens, 2000).
- Conjoint analysis (Luce and Tukey).
- Admissible Geometrization™, a hybrid approach having classical characteristics, interpretable as optimal embedding of admissibly weighted complete graphs.

Meaningful Ontology Construction

- Creating an ontology or description for the local dynamical structure of the firm may require measurement theoretically meaningful approaches to analysis and synthesis/fusion of intermixed scale type databases.
- In the absence of ground truth, meaningful manipulation of known and emerging indexicals and indices may be important for constructing robust and consistent managerial ontologies.
- Automation/simulation over emerging indexicals and indices may require control, commensuration, conversion of intermixed scale type data.

Some Open Problems

- Quantify/demonstrate the effects of meaningfulness and meaninglessness for real-world data.
- Demonstrate that meaningful methodologies provide a competitive advantage for the firm through more accurate and/or less arbitrary results.
- Elucidate the relationship between uncertainty, error, scale types, and meaningfulness.
- Understand and apply meaningfulness to risk analysis and decision support.
- Does meaningfulness matter for managerial ontology construction? If not, why not?

Appendix: Relational Systems

A *relational system* or *structure* is a finite sequence of the form

$$\mathrm{U} = \langle A, R_1, \ldots, R_n \rangle = \langle A, R_i \rangle_{i \in I}$$

where A is a finite set called the *domain* of the relational system and the R_i, $i \in I = \{1, \ldots, n\}$ are relations on A.

Let $\mathfrak{A} = \langle A, R_i \rangle_{i \in I}$ and $\mathfrak{B} = \langle B, S_j \rangle_{j \in J}$ be similar relational systems, then a *homomorphism* f from $\mathfrak{A}$ into $\mathfrak{B}$ is a function f from A into B such that for all $i \in I$ and elements $a_1, \ldots, a_{m(i)}$ in A,

$$R_i\left(a_1, \ldots, a_{m(i)}\right) \quad \text{iff} \quad S_i\left(f\left(a_1\right), \ldots, f\left(a_{m(i)}\right)\right)$$

Scales and Meaningfulness

Let ✞ and ☠ be relational systems where the structure ☠ is based on a subset of the real numbers (e.g. $\mathbf{R}^+$), then a (structured) *scale* is the set Hom(✞,☠) of homomorphisms from ✞ into ☠.

A group G is a *scale group* for a scale Hom(✞,☠) if G acts on Im ✞ = { f (x) | x ∈ A and f ∈ Hom(✞,☠) } through composition of functions and there exists an h ∈ Hom(✞,☠) such that Hom(✞,☠) = G_h = { g ⬆ h | g ∈ G }.

Let ☠ be a numerical system based on $\mathbf{R}^+$, then an n–ary relation T on $\mathbf{R}^+$ is *quantitatively* ☠–meaningful for structure ✞ = ↑A, R_i⇧$_{i∈I}$ if for all ℮Ƭ, ⊠ ∈ Hom(✞,☠),

$$T\left(\varphi\left(a_1\right),\ldots,\varphi\left(a_n\right)\right)=T\left(\psi\left(a_1\right),\ldots,\psi\left(a_n\right)\right)$$

for all $a_1,\ldots,a_n$ ∈ A.

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